Documentation of the FredsEmpirical formula for estimating travel times from route distance

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Abstract

The FredsEmpirical formula is a linear function of route distance, cumulative elevation gain and terrain type that estimates the backpacking travel time. This document provides the data sources and statistical regression model for the FredsEmpirical formula.

$\boldsymbol{x}_i, i = 0, \ldots, n$ Successive positions listed in test route.

$d(\boldsymbol{x}, \boldsymbol{y})$ A function that returns the great circle distance between two positions.

$C_i, i = 1, \ldots, n$ A list of binary variables, $C_i = 1$ if the corresponding segment is over cross country, and $C_i = 0$ is on trail.

$S = \sum_{i=1}^{n} d(\boldsymbol{x}_i, \boldsymbol{x}_{i-1})(1 - C_i)$ Total estimated trail distance of test route.

$R = \sum_{i=1}^{n} d(\boldsymbol{x}_i, \boldsymbol{x}_{i-1}) C_i$ Total estimated cross country distance of test route.

$z_i, i = 0, \ldots, n$ Elevation estimates for each position in test route.

$Z = \sum_{i=1}^{n} \max(0, z_i - z_{i-1})$ Cumulative elevation gain for test route.

τ_0, τ_f Time stamps of the start and finish times for actual backpack of test route.

$T = \tau_f - \tau_0$ Meaured travel time for test route.

Background and problem statement

Good planning of backpack trips includes knowing how far you can go in one day. Factors that affect backpacking travel times are distance traveled, elevation gain, roughness of the trail, amount of weight carried, overall conditioning of the backpacker and the motivation of the backpacker. In spite of all of these unknowns, I can use an empirical estimator of backpack travel time based on length of the route, the elevation gain and the roughness of the trail.

When I was a young scout, there was a widely known "folk formula" that went like this: allow for two miles an hour travel plus an extra hour for every 1,000 feet of climbing. This formula is conservative enough to keep most backpackers out of trouble. It worked for scouts of 11 years age and up. As an adult, I found I could substantially beat the uphill time estimate. However, if the trail was especially rugged, I would fall a little behind on the downhill side.

Cross country routes generally (not always) take longer than trail. Cross country travel adds such a level of uncertainty, that that backpack planner needs to estimate a margin for uncertainty as well as an expected travel time.

Ways of measuring distance and elevation gain have been the same since I was a young scout. They have just gotten much more efficient and reliable with computerized tools. A route is a manually drawn polygon of positions, $\boldsymbol{x}_i, i = 0, \ldots, n$, over a map. The estimated distance for a route is the sum of the lengths of the polygon segments, $S = \sum_{i=1}^{n} d\left(\boldsymbol{x}_i, \boldsymbol{x}_{i-1}\right)$. The cumulative elevation gain of the route is the sum of the elevation gains of the polygon segments, $Z = \sum_{i=1}^{n} \max\left(0, z_i - z_{i-1}\right)$, the order of the indices taken in the direction that the backpacker plans to travel the route. If $z_i < z_{i-1}$,then the elevation gain is zero for that segment.

I am going to make a distinction between a route and a track. While a route is manually drawn from a map or mapping software, a track is a recording made by a geolocation device, such as GPS. A track may have hundreds of positions recorded, while only tens of points is practical for a route. A track is affected by real-world factors that are not visible to the planner using only a map. Therefore, the FredsEmpirical formula can only reliably be used on data measured from routes.

The problem is how to predict the travel time for a route when the input variables known for the route are miles on trail, miles cross country and cumulative elevation gain. Routes will constitute about a day or more of travel time, the whole point of the estimation formula being to determine how far one should go before camping. The travel time formula shall include rest time as well as time actually hiking.

Model of backpacking route travel time

The statistical model for the route travel time is

$$T = \beta_S S + \beta_R R + \beta_Z Z + \varepsilon,$$

where T is the travel time in hours, S is the trail distance , R is the cross country distance, Z is the cumulative elevation of the route, ε is the prediction error and sum of all uncertain effects and $\beta_S, \beta_R, \beta_Z$ are the unknown regression coefficients.

$$S = \sum_{i=1}^{n} d\left(\boldsymbol{x}_i, \boldsymbol{x}_{i-1}\right)\left(1 - C_i\right)$$

$$R = \sum_{i=1}^{n} d\left(\boldsymbol{x}_i, \boldsymbol{x}_{i-1}\right) C_i$$

$$Z = \sum_{i=1}^{n} \max\left(0, z_i - z_{i-1}\right).$$

Route travel time data

I collected travel time, trail distance, cross country distance and elevation gain for 31 days of backpacking spread over four backpacking trips.

trip	segment	miles_trail	miles_CC	elev_gain_feet	total hours
mammoth crest	0	3.26	0	1437	2.28
mammoth crest	1	9.21	0	2776	7.13
mammoth crest	2	0	3.86	1863	7.65
mammoth crest	3	0	2.74	1611	6.93
mammoth crest	4	0	2.5	1393	6.82
mammoth crest	5	4.1	5.71	2619	11.08
mammoth crest	6	2.49	1.47	2222	8.26
mammoth crest	7	8.59	1.39	1969	5.88
brewer loop	0	4.41	0	1884	2.64
brewer loop	1	3.47	0	2722	4.01
brewer loop	2	0	2.91	2180	5.4
brewer loop	3	0	1	495	1.01
brewer loop	4	0	2.21	1225	4.3
brewer loop	5	0	2.57	1431	4.38
brewer loop	6	0	2.29	1049	3.69
brewer loop	7	0	0.66	104	0.65
brewer loop	8	1.35	0.92	471	2.7
brewer loop	9	2.72	0	219	1.41
brewer loop	10	10.54	0	915	5.08
abbot loop	0	3.7	0	2747	3.63
abbot loop	1	2.09	2.26	2238	6.45
abbot loop	2	0	3.88	1401	6.02
abbot loop	3	0	3.75	1228	5.71
abbot loop	4	7.63	2.01	2197	8.73
abbot loop	5	6.06	0	2406	6.24
abbot loop	6	0	0.97	311	0.57
abbot loop	7	1.91	0	470	2.17
abbot loop	8	6.46	0	303	3.46
rock island lake loop	3	0	2.30	961	5.2
rock island lake loop	4	0.72	1.53	377	3
rock island lake loop	5	0	3.35	412	6.2